

Note on weight-monodromy conjecture for p -adically uniformized varieties

Yoichi Mieda

ABSTRACT. We prove the weight-monodromy conjecture for varieties which are p -adically uniformized by a product of the Drinfeld upper half spaces. It is an easy consequence of Dat's work on the cohomology complex of the Drinfeld upper half space.

1 Introduction

Let X be a proper smooth variety over a p -adic field F . For a prime number $\ell \neq p$ and an integer i , the absolute Galois group $\text{Gal}(\overline{F}/F)$ acts on the i th ℓ -adic étale cohomology $H^i(X \otimes_F \overline{F}, \overline{\mathbb{Q}}_\ell)$. This action determines two filtrations on the cohomology; the weight filtration and the monodromy filtration. The weight-monodromy conjecture predicts that these two filtrations coincide up to shift by i . This conjecture, due to Deligne [Del71], is widely open. It is known in the following cases:

- (i) X has good reduction over \mathcal{O}_F ([Del74], [Del80]).
- (ii) X is an abelian variety ([SGA7, Exposé IX]).
- (iii) $i \leq 2$ ([RZ82], [dJ96]).
- (iv) X is uniformized by the covering of the Drinfeld upper half space ([Ito05], [Dat06], [Dat07]).
- (v) X is a set-theoretic complete intersection in a projective smooth toric variety ([Sch12]).

In this short note, we will slightly generalize the case (iv); we will consider a variety X which is uniformized by a product of the Drinfeld upper half spaces.

Our setting is as follows. Let F, F' be p -adic fields and F'' a p -adic field containing F and F' . Fix integers $d, d' \geq 1$ and put $G = \text{PGL}_d(F)$, $G' = \text{PGL}_{d'}(F')$, respectively. Let $\Omega_F = \Omega_F^{d-1}$ (resp. $\Omega_{F'} = \Omega_{F'}^{d'-1}$) denote the $d-1$ -dimensional (resp. $d'-1$ -dimensional) Drinfeld upper half space. To simplify the notation, we write $\Omega_F \times_{F''} \Omega_{F'} = (\Omega_F \otimes_F F'') \times_{F''} (\Omega_{F'} \otimes_{F'} F'')$. For a discrete torsion-free cocompact

Graduate School of Mathematical Sciences, The University of Tokyo, 3-8-1 Komaba, Meguro-ku, Tokyo, 153-8914, Japan

E-mail address: mieda@ms.u-tokyo.ac.jp

2010 *Mathematics Subject Classification*. Primary: 11G25; Secondary: 11F70, 22E50.

subgroup $\Gamma \subset G \times G'$, the quotient $\Omega_F \times_{F''} \Omega_{F'}/\Gamma$ becomes a projective smooth variety over F'' . Such a variety is said to be uniformized by $\Omega_F \times_{F''} \Omega_{F'}$.

The main theorem of this article is the following:

Theorem 1.1 *Let X be a projective smooth variety over F'' which is uniformized by $\Omega_F \times_{F''} \Omega_{F'}$. Then, the weight-monodromy conjecture holds for X .*

Our strategy is the same as that in [Dat06]. First we determine the monodromy operator on the cohomology complex $R\Gamma_c((\Omega_F \times_{F''} \Omega_{F'}) \otimes_{F''} \overline{F''}, \overline{\mathbb{Q}}_\ell)$, which is an object of the derived category of smooth $G \times G'$ -representations. Using this result, one can easily compute the cohomology of X , from which the weight monodromy conjecture is deduced.

Although Theorem 1.1 is stated for the product of two Drinfeld upper half spaces, our argument also works for the product of more than two Drinfeld upper half spaces. Further, as in [Dat07], we may replace the Drinfeld upper half space by its covering introduced by Drinfeld [Dri76]. See Theorem 2.5.

Interesting examples of varieties uniformized by a product of Drinfeld upper half spaces are given by some unitary Shimura varieties (see [RZ96, Theorem 6.50]). By using our result, we can compute the ℓ -adic cohomology and the local Hasse-Weil zeta functions of such Shimura varieties without any effort. See the remark in the end of this note.

Acknowledgment This work was supported by JSPS KAKENHI Grant Number 24740019.

Notation Throughout this paper, ℓ denotes a prime number different from p . Fix an isomorphism $\overline{\mathbb{Q}}_\ell \cong \mathbb{C}$ and identify them. All representations are considered over this field.

In the notation of ℓ -adic étale cohomology, we omit the coefficient $\overline{\mathbb{Q}}_\ell$ and the base change to a separable closure. For example, $H^i(X \otimes_F \overline{F}, \overline{\mathbb{Q}}_\ell)$ is written as $H^i(X)$. As in Introduction, let F, F' and F'' be p -adic fields with $F, F' \subset F''$. For a rigid space X (resp. X') over F (resp. F'), we write $X \times_{F''} X'$ for $(X \otimes_F F'') \otimes_{F''} (X' \otimes_{F'} F'')$.

2 Proof

For a subset I of $\{1, \dots, d-1\}$, an irreducible smooth representation π_I of G is naturally attached (see [Dat06, 2.1.3]). For example, π_\emptyset is the Steinberg representation \mathbf{St}_d and $\pi_{\{1, \dots, d-1\}}$ the trivial representation $\mathbf{1}$. For $0 \leq i \leq d-1$, we write $\pi_{\leq i}$ for $\pi_{\{1, \dots, i\}}$. Similarly, for $J \subset \{1, \dots, d'-1\}$, consider an irreducible smooth representation π'_J of G' .

Lemma 2.1 (i) *For $I_1, I_2 \subset \{1, \dots, d-1\}$ and $J_1, J_2 \subset \{1, \dots, d'-1\}$, we have*

$$\mathrm{Ext}_{G \times G'}^i(\pi_{I_1} \boxtimes \pi'_{J_1}, \pi_{I_2} \boxtimes \pi'_{J_2}) = \begin{cases} \overline{\mathbb{Q}}_\ell & \text{if } i = \delta(I_1, I_2) + \delta(J_1, J_2), \\ 0 & \text{otherwise.} \end{cases}$$

Here $\delta(I_1, I_2) = \#(I_1 \cup I_2) - \#(I_1 \cap I_2)$.

Weight-monodromy conjecture for p -adically uniformized varieties

- (ii) Let I_1, I_2, I_3 be subsets of $\{1, \dots, d-1\}$ satisfying $\delta(I_1, I_2) + \delta(I_2, I_3) = \delta(I_1, I_3)$. Take a non-zero element $\beta \in \text{Ext}_G^{\delta(I_1, I_2)}(\pi_{I_1}, \pi_{I_2})$. For $J_1, J_2 \subset \{1, \dots, d'-1\}$, the homomorphism

$$\begin{aligned} & \text{Ext}_{G \times G'}^{\delta(I_2, I_3) + \delta(J_1, J_2)}(\pi_{I_2} \boxtimes \pi'_{J_1}, \pi_{I_3} \boxtimes \pi'_{J_2}) \\ & \xrightarrow{(\beta \cup -) \boxtimes \text{id}} \text{Ext}_{G \times G'}^{\delta(I_1, I_3) + \delta(J_1, J_2)}(\pi_{I_1} \boxtimes \pi'_{J_1}, \pi_{I_3} \boxtimes \pi'_{J_2}) \end{aligned}$$

is an isomorphism.

- (iii) Let $\pi \boxtimes \pi'$ be an irreducible smooth representation of $G \times G'$. If it is not of the form $\pi_{I_0} \boxtimes \pi'_{J_0}$ with $I_0 \subset \{1, \dots, d-1\}$ and $J_0 \subset \{1, \dots, d'-1\}$, then we have $\text{Ext}_{G \times G'}^i(\pi_I \boxtimes \pi'_J, \pi \boxtimes \pi') = 0$ for every $I \subset \{1, \dots, d-1\}$, $J \subset \{1, \dots, d'-1\}$ and i .

Proof. For (i) and (ii), apply [Dat06, Théorème 1.3] to the semisimple group $G \times G'$. The claim (iii) follows from [Vig97, Theorem 6.1], since the cuspidal supports of $\pi_I \boxtimes \pi'_J$ and $\pi \boxtimes \pi'$ are different. \blacksquare

Now we recall a result of Dat, which is crucial for our work. In [Dat06], he studied the cohomology complex $R\Gamma_c(\Omega_F) = R\Gamma_c(\Omega_F \otimes_F \overline{F}, \overline{\mathbb{Q}}_\ell)$, which is an object of the bounded derived category of smooth representations of G . The Weil group W_F of F acts on $R\Gamma_c(\Omega_F)$.

Theorem 2.2 ([Dat06]) *Fix a Frobenius lift $\varphi \in W_F$.*

- (i) ([Dat06, Proposition 4.2.2]) *There exists a unique isomorphism*

$$\alpha: R\Gamma_c(\Omega_F) \xrightarrow{\cong} \bigoplus_{i=0}^{d-1} \pi_{\leq i}(-i)[-d+1-i]$$

compatible with the actions of φ . It induces an isomorphism

$$\text{End}(R\Gamma_c(\Omega_F)) \cong \bigoplus_{0 \leq i < j \leq d-1} \text{Ext}_G^{j-i}(\pi_{\leq j}, \pi_{\leq i})(j-i).$$

- (ii) ([Dat06, Lemme 4.2.1]) *The monodromy operator $N \in \text{End}(R\Gamma_c(\Omega_F))(-1)$ on $R\Gamma_c(\Omega_F)$ is naturally determined. The image of N under the isomorphism in (i) belongs to $\bigoplus_{0 \leq i \leq d-2} \text{Ext}_G^1(\pi_{\leq i+1}, \pi_{\leq i})$. We denote it by $(\beta_i)_i$.*
- (iii) ([Dat06, Proposition 4.2.7]) *For each i with $0 \leq i \leq d-2$, $\beta_i \neq 0$.*

The following theorem is an analogue of [Dat06, Théorème 1.1].

Theorem 2.3 *For subsets $I \subset \{1, \dots, d-1\}$ and $J \subset \{1, \dots, d'-1\}$, we have an isomorphism of Weil-Deligne representations of F'' :*

$$\mathcal{H}^*(R\text{Hom}(R\Gamma_c(\Omega_F \times_{F''} \Omega_{F'}), \pi_I \boxtimes \pi'_J)) \cong \text{rec}_F(\pi_I)(\frac{d-1}{2})|_{W_{F''}} \otimes \text{rec}_{F'}(\pi'_J)(\frac{d'-1}{2})|_{W_{F''}},$$

where rec_F (resp. $\text{rec}_{F'}$) denotes the local Langlands correspondence for F (resp. F'). The functor \mathcal{H}^* from $D^b(\overline{\mathbb{Q}}_\ell)$ to the category of \mathbb{Z} -graded $\overline{\mathbb{Q}}_\ell$ -vector spaces is given by $L^\bullet \mapsto \bigoplus_{i \in \mathbb{Z}} H^i(L^\bullet)$.

If an irreducible smooth representation $\pi \boxtimes \pi'$ of $G \times G'$ is not of the form $\pi_I \boxtimes \pi'_J$, then $R \text{Hom}(R\Gamma_c(\Omega_F \times_{F''} \Omega_{F'}), \pi \boxtimes \pi') = 0$.

Proof. For simplicity, we only consider the cases $(I, J) = (\emptyset, \emptyset), (\emptyset, \{1, \dots, d' - 1\})$. Other cases can be treated similarly.

First consider the case $(I, J) = (\emptyset, \emptyset)$. By Theorem 2.2 (i) and the Künneth formula, we have

$$R\Gamma_c(\Omega_F \times_{F''} \Omega_{F'}) \xrightarrow{\cong} \bigoplus_{i=0}^{d-1} \bigoplus_{j=0}^{d'-1} (\pi_{\leq i} \boxtimes \pi'_{\leq j})(-i-j)[-d-d'+2-i-j].$$

By Lemma 2.1 (i), we have

$$R \text{Hom}(R\Gamma_c(\Omega_F \times_{F''} \Omega_{F'}), \pi_\emptyset \boxtimes \pi'_\emptyset) \cong \bigoplus_{i=0}^{d-1} \bigoplus_{j=0}^{d'-1} \text{Ext}_{G \times G'}^{i+j}(\pi_{\leq i} \boxtimes \pi'_{\leq j}, \pi_\emptyset \boxtimes \pi'_\emptyset)(i+j),$$

where $\text{Ext}_{G \times G'}^{i+j}(\pi_{\leq i} \boxtimes \pi'_{\leq j}, \pi_\emptyset \boxtimes \pi'_\emptyset)$ is a one-dimensional vector space for each i and j . Let $e_{0,0} \in \text{Hom}_{G \times G'}(\pi_\emptyset \boxtimes \pi'_\emptyset, \pi_\emptyset \boxtimes \pi'_\emptyset)$ be the identity. Define $e_{i,j} \in \text{Ext}_{G \times G'}^{i+j}(\pi_{\leq i} \boxtimes \pi'_{\leq j}, \pi_\emptyset \boxtimes \pi'_\emptyset)$ as the image of $e_{0,0}$ under the map

$$(\beta_{i-1} \cup \dots \cup \beta_0 \cup -) \boxtimes (\beta'_{j-1} \cup \dots \cup \beta'_0 \cup -).$$

Here, $(\beta'_j) \in \bigoplus_{0 \leq j \leq d'-2} \text{Ext}_{G'}^1(\pi'_{j+1}, \pi'_j)$ denotes the image of $N \in \text{End}(R\Gamma_c(\Omega_{F'}))(-1)$. By Lemma 2.1 (ii) and Theorem 2.2 (iii), $e_{i,j}$ is a basis of $\text{Ext}_{G \times G'}^{i+j}(\pi_{\leq i} \boxtimes \pi'_{\leq j}, \pi_\emptyset \boxtimes \pi'_\emptyset)$. Now the monodromy operator on $R \text{Hom}(R\Gamma_c(\Omega_F \times_{F''} \Omega_{F'}), \pi_\emptyset \boxtimes \pi'_\emptyset)$ can be described explicitly:

$$Ne_{i,j} = e_{i+1,j} + e_{i,j+1}.$$

Therefore we conclude that

$$\begin{aligned} \mathcal{H}^*(R \text{Hom}(R\Gamma_c(\Omega_F \times_{F''} \Omega_{F'}), \pi_\emptyset \boxtimes \pi'_\emptyset)) &\cong \mathbf{Sp}_d(\frac{d-1}{2}) \otimes \mathbf{Sp}_{d'}(\frac{d'-1}{2}) \\ &= \text{rec}_F(\pi_\emptyset)(\frac{d-1}{2})|_{W_{F''}} \otimes \text{rec}_{F'}(\pi'_\emptyset)(\frac{d'-1}{2})|_{W_{F''}}. \end{aligned}$$

We note that the right hand side concentrates in the degree $-d-d'+2$.

Next assume that $I = \emptyset$ and $J = \{1, \dots, d' - 1\}$. Then, Lemma 2.1 (i) tells us that

$$\begin{aligned} R \text{Hom}(R\Gamma_c(\Omega_F \times_{F''} \Omega_{F'}), \pi_\emptyset \boxtimes \pi'_{\leq d'-1}) \\ \cong \bigoplus_{i=0}^{d-1} \bigoplus_{j=0}^{d'-1} \text{Ext}_{G \times G'}^{i+d'-1-j}(\pi_{\leq i} \boxtimes \pi'_{\leq j}, \pi_\emptyset \boxtimes \pi'_{\leq d'-1})(i+j). \end{aligned}$$

The (i, j) -component on the right hand side has degree $-d + 1 - 2j$. Note that $\text{id} \boxtimes (\beta'_j \cup -)$ induces the zero map on the right hand side. On the other hand, $(\beta_i \cup -) \boxtimes \text{id}$ gives an isomorphism

$$\text{Ext}_{G \times G'}^{i+d'-1-j}(\pi_{\leq i} \boxtimes \pi'_{\leq j}, \pi_{\emptyset} \boxtimes \pi'_{\leq d'-1}) \longrightarrow \text{Ext}_{G \times G'}^{i+d'-j}(\pi_{\leq i+1} \boxtimes \pi'_{\leq j}, \pi_{\emptyset} \boxtimes \pi'_{\leq d'-1}).$$

Therefore, $\mathcal{H}^*(R\text{Hom}(R\Gamma_c(\Omega_F \times_{F''} \Omega_{F'}), \pi_{\emptyset} \boxtimes \pi'_{\leq d'-1}))$ is isomorphic to

$$\begin{aligned} \mathbf{Sp}_d\left(\frac{d-1}{2}\right) \otimes (\overline{\mathbb{Q}}_{\ell} \oplus \overline{\mathbb{Q}}_{\ell}(1) \oplus \cdots \oplus \overline{\mathbb{Q}}_{\ell}(d'-1)) \\ = \text{rec}_F(\pi_{\emptyset})\left(\frac{d-1}{2}\right)|_{W_{F''}} \otimes \text{rec}_{F'}(\pi'_{\leq d'-1})\left(\frac{d'-1}{2}\right)|_{W_{F''}}. \end{aligned}$$

Finally, if $\pi \boxtimes \pi'$ is not of the form $\pi_I \boxtimes \pi'_J$,

$$\begin{aligned} R\text{Hom}(R\Gamma_c(\Omega_F \times_{F''} \Omega_{F'}), \pi \boxtimes \pi') \\ = \bigoplus_{i=0}^{d-1} \bigoplus_{j=0}^{d'-1} R\text{Hom}(\pi_{\leq i} \boxtimes \pi'_{\leq j}, \pi \boxtimes \pi')(i+j)[d+d'-2+i+j] = 0 \end{aligned}$$

by Lemma 2.1 (iii). ■

Corollary 2.4 *Let Γ be a discrete torsion-free cocompact subgroup of $G \times G'$. Let $m_{1,0}$ (resp. $m_{0,1}$, resp. $m_{1,1}$) be the multiplicity of $\mathbf{St}_d \boxtimes \mathbf{1}$ (resp. $\mathbf{1} \boxtimes \mathbf{St}_{d'}$, resp. $\mathbf{St}_d \boxtimes \mathbf{St}_{d'}$) in the representation $C^\infty(G \times G'/\Gamma)$ of $G \times G'$. Then, we have a $W_{F''}$ -isomorphism*

$$\begin{aligned} R\Gamma(\Omega_F \times_{F''} \Omega_{F'}/\Gamma) \cong & \left(\bigoplus_{i=0}^{d-1} \bigoplus_{j=0}^{d'-1} \overline{\mathbb{Q}}_{\ell}(-i-j)[-2i-2j] \right) \\ & \oplus \left(\mathbf{Sp}_d\left(\frac{1-d}{2}\right)[1-d] \otimes \left(\bigoplus_{j=0}^{d'-1} \overline{\mathbb{Q}}_{\ell}(-j)[-2j] \right) \right)^{m_{1,0}} \\ & \oplus \left(\left(\bigoplus_{i=0}^{d-1} \overline{\mathbb{Q}}_{\ell}(-i)[-2i] \right) \otimes \mathbf{Sp}_{d'}\left(\frac{1-d'}{2}\right)[1-d'] \right)^{m_{0,1}} \\ & \oplus (\mathbf{Sp}_d \otimes \mathbf{Sp}_{d'})^{m_{1,1}}\left(\frac{2-d-d'}{2}\right)[2-d-d']. \end{aligned}$$

Moreover, the weight-monodromy conjecture holds for $\Omega_F \times_{F''} \Omega_{F'}/\Gamma$.

Proof. The proof is the same as [Dat06, Corollaire 4.5.1]. By the fixed isomorphism $\overline{\mathbb{Q}}_{\ell} \cong \mathbb{C}$, we regard $C^\infty(G \times G'/\Gamma)$ as a representation over \mathbb{C} . Then, it is a unitary representation, and decomposes into the direct sum of irreducible smooth unitary representations of $G \times G'$ with finite multiplicities. Assume that $\pi_I \boxtimes \pi'_J$ appears in $C^\infty(G \times G'/\Gamma)$. Then it is unitary, and thus π_I and π'_J are unitary. Hence we conclude that I (resp. J) is either \emptyset or $\{1, \dots, d-1\}$ (resp. $\{1, \dots, d'-1\}$). Note that the multiplicity of the trivial representation $\mathbf{1} \boxtimes \mathbf{1}$ in $C^\infty(G \times G'/\Gamma)$ equals 1.

As in the proof of [Dat06, Corollaire 4.5.1], we have

$$\begin{aligned}
R\Gamma(\Omega_F \times_{F''} \Omega_{F'}/\Gamma)^\vee &\cong (R\Gamma_c(\Omega_F \times_{F''} \Omega_{F'}) \overset{\mathbb{L}}{\otimes}_{\overline{\mathbb{Q}_\ell}[\Gamma]} \overline{\mathbb{Q}_\ell})^\vee \\
&\cong R\mathrm{Hom}(R\Gamma_c(\Omega_F \times_{F''} \Omega_{F'}), C^\infty(G \times G'/\Gamma)) \\
&= R\mathrm{Hom}(R\Gamma_c(\Omega_F \times_{F''} \Omega_{F'}), (\mathbf{1} \boxtimes \mathbf{1}) \oplus (\mathbf{St}_d \boxtimes \mathbf{1})^{m_{1,0}} \\
&\quad \oplus (\mathbf{1} \boxtimes \mathbf{St}_{d'})^{m_{0,1}} \oplus (\mathbf{St}_d \boxtimes \mathbf{St}_{d'})^{m_{1,1}}).
\end{aligned}$$

By using Theorem 2.3 and taking dual, we obtain the desired description of $R\Gamma(\Omega_F \times_{F''} \Omega_{F'}/\Gamma)$.

For the weight-monodromy conjecture, just note that

$$\mathbf{Sp}_d \otimes \mathbf{Sp}_{d'} \cong \bigoplus_{\substack{|d-d'| \leq j \leq d+d', \\ j \equiv d+d' \pmod{2}}} \mathbf{Sp}_j$$

(see [Del80, (1.6.11.2)]). ■

The argument above applies to the product of more than two Drinfeld upper half spaces without any difficulty. Furthermore, it is also valid even if we replace the Drinfeld upper half spaces by its coverings introduced in [Dri76]. Namely, the following theorem holds.

Theorem 2.5 *Let D (resp. D') be the central division algebra over F (resp. F') with invariant $1/d$ (resp. $1/d'$). We denote $\mathcal{M} = \{\mathcal{M}_n\}$ (resp. $\mathcal{M}' = \{\mathcal{M}'_{n'}\}$) the Drinfeld tower on which D^\times (resp. D'^\times) acts.*

- (i) *Fix irreducible smooth representations ρ, ρ' of D^\times, D'^\times , respectively. Let π (resp. π') be an irreducible smooth representation of $\mathrm{GL}_n(F)$ (resp. $\mathrm{GL}_{n'}(F')$) with the same central character as ρ (resp. ρ').*
- (a) *If $\rho = \mathrm{LJ}_d(\pi)$ and $\rho' = \mathrm{LJ}_{d'}(\pi')$ (for the definition of LJ , see [Dat07, §2]), then we have*

$$\begin{aligned}
\mathcal{H}^*(R\mathrm{Hom}(R\Gamma_c(\mathcal{M} \times_{F''} \mathcal{M}')[\rho \boxtimes \rho']), \pi \boxtimes \pi') \\
\cong \mathrm{rec}_F(\pi)(\frac{d-1}{2})|_{W_{F''}} \oplus \mathrm{rec}_{F'}(\pi')(\frac{d'-1}{2})|_{W_{F''}}.
\end{aligned}$$

- (b) *Otherwise $R\mathrm{Hom}(R\Gamma_c(\mathcal{M} \times_{F''} \mathcal{M}')[\rho \boxtimes \rho']), \pi \boxtimes \pi' = 0$.*

(See also [Dat07, Lemme 4.4.1].)

- (ii) *Let Γ be a discrete torsion-free cocompact subgroup of $\mathrm{GL}_d(F) \times \mathrm{GL}_{d'}(F')$, and $n, n' \geq 0$ integers. Then, $R\Gamma(\mathcal{M}_n \times_{F''} \mathcal{M}'_{n'}/\Gamma)$ can be computed as in [Dat07, p. 139–140]. In particular, the weight-monodromy conjecture holds for $\mathcal{M}_n \times_{F''} \mathcal{M}'_{n'}/\Gamma$.*

Proof. Use the result in [Dat07] in place of Theorem 2.2. ■

We may apply Theorem 2.5 to the unitary Shimura varieties appearing in [RZ96, Theorem 6.50]. By the same method as in [She14, §3], one can compute the ℓ -adic cohomology of them using Theorem 2.5 (i). This considerably simplifies the proof of the main result of [She14]; the study on test functions in [She14, §4–7] is no longer needed. The local Hasse-Weil zeta functions of such Shimura varieties can be computed directly. The result is the same as in [She14, Corollary 7.4] (but we do not need the assumption $r = 1$).

References

- [Dat06] J.-F. Dat, *Espaces symétriques de Drinfeld et correspondance de Langlands locale*, Ann. Sci. École Norm. Sup. (4) **39** (2006), no. 1, 1–74.
- [Dat07] ———, *Théorie de Lubin-Tate non-abélienne et représentations elliptiques*, Invent. Math. **169** (2007), no. 1, 75–152.
- [Del71] P. Deligne, *Théorie de Hodge. I*, Actes du Congrès International des Mathématiciens (Nice, 1970), Tome 1, Gauthier-Villars, Paris, 1971, pp. 425–430.
- [Del74] ———, *La conjecture de Weil. I*, Inst. Hautes Études Sci. Publ. Math. (1974), no. 43, 273–307.
- [Del80] ———, *La conjecture de Weil. II*, Inst. Hautes Études Sci. Publ. Math. (1980), no. 52, 137–252.
- [dJ96] A. J. de Jong, *Smoothness, semi-stability and alterations*, Inst. Hautes Études Sci. Publ. Math. (1996), no. 83, 51–93.
- [Dri76] V. G. Drinfeld, *Coverings of p -adic symmetric domains*, Funkcional. Anal. i Priložen. **10** (1976), no. 2, 29–40.
- [Ito05] T. Ito, *Weight-monodromy conjecture for p -adically uniformized varieties*, Invent. Math. **159** (2005), no. 3, 607–656.
- [RZ82] M. Rapoport and Th. Zink, *Über die lokale Zetafunktion von Shimuravarietäten. Monodromiefiltration und verschwindende Zyklen in ungleicher Charakteristik*, Invent. Math. **68** (1982), no. 1, 21–101.
- [RZ96] ———, *Period spaces for p -divisible groups*, Annals of Mathematics Studies, vol. 141, Princeton University Press, Princeton, NJ, 1996.
- [Sch12] P. Scholze, *Perfectoid spaces*, Publ. Math. Inst. Hautes Études Sci. **116** (2012), no. 1, 245–313.
- [She14] X. Shen, *On the ℓ -adic cohomology of some p -adically uniformized shimura varieties*, preprint, [arXiv:1411.0244v1](https://arxiv.org/abs/1411.0244v1), 2014.
- [Vig97] M.-F. Vignéras, *Extensions between irreducible representations of a p -adic $GL(n)$* , Pacific J. Math. (1997), no. Special Issue, 349–357, Olga Taussky-Todd: in memoriam.
- [SGA7] *Groupes de monodromie en géométrie algébrique (SGA7)*, Lecture Notes in Mathematics, Vol. 288, 340, Springer-Verlag, Berlin, 1972.